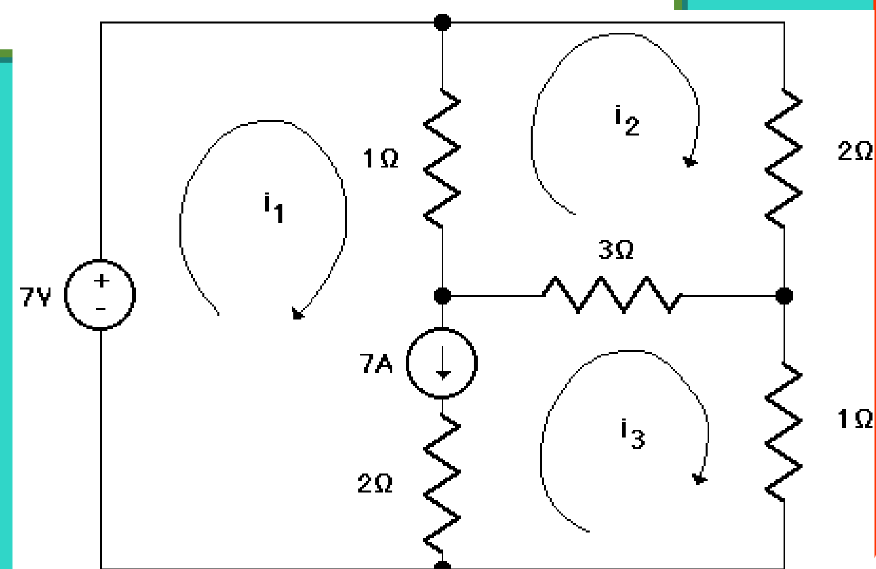
Chapter 26

Current and Resistance



İZMİR KÂTİP ÇELEBİ ÜNİVERSİTESİ

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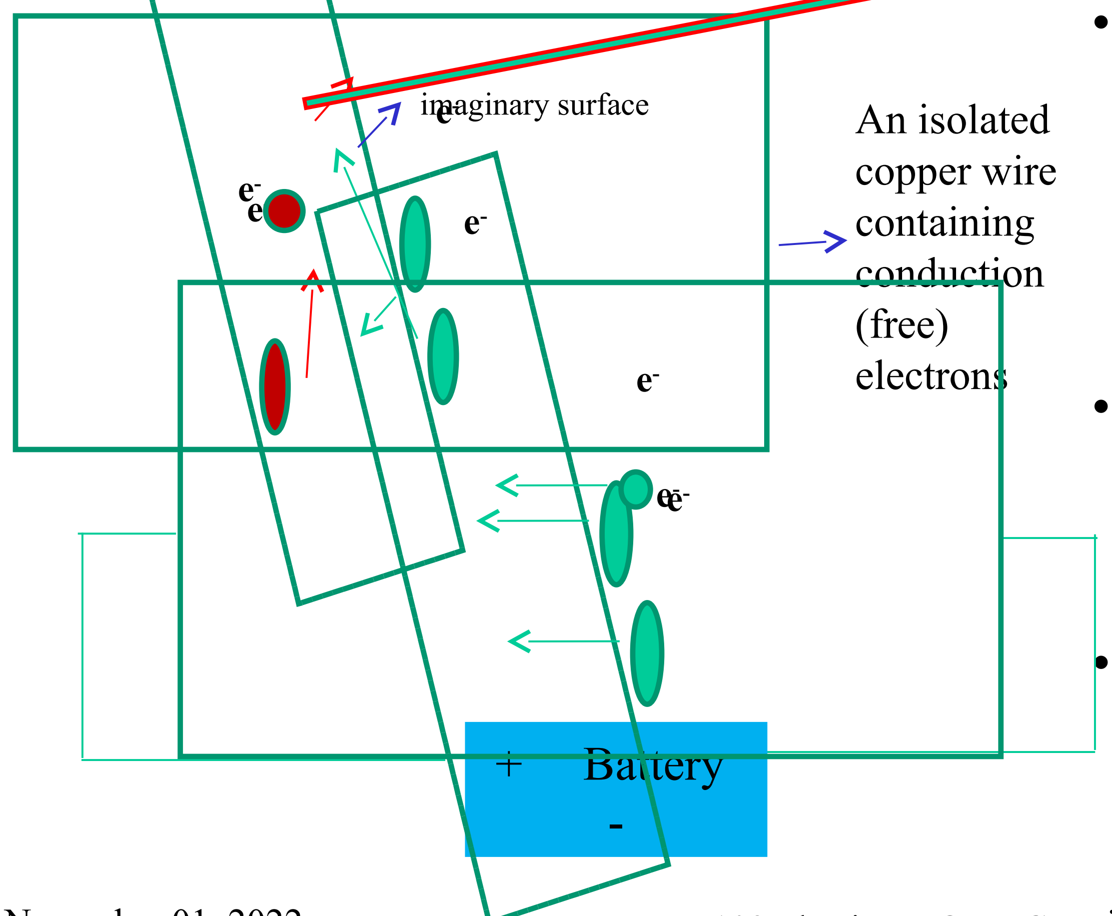
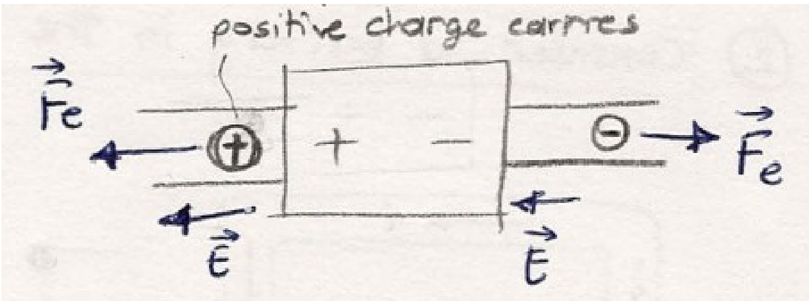
- Current
- Conservation of Charge
- Current Density, J
- Drift Speed, V_d
- Resistance
- Resistivity ρ & Conductivity σ
- Macroscopic & Microscopic Expressions

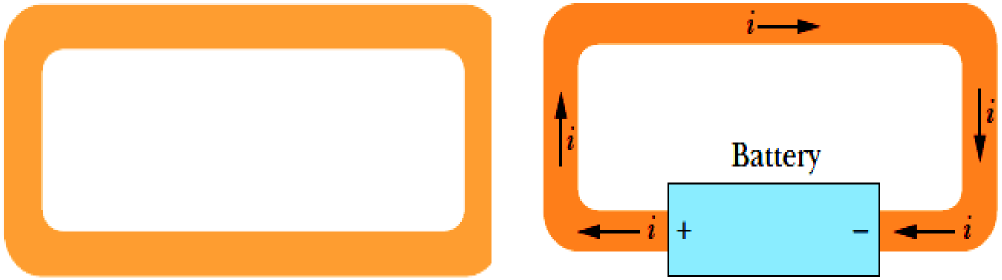
- **An electric current is a stream of moving charges**
  - *Not all moving charges constitute an electric current.*
  - If there is to be an electric current through a given surface, there must be **a net flow of charge** through that surface.
- Free electrons (conduction electrons) are moving *in random motion* at speeds of order 10^6 m/s.



- Conduction electrons pass through both directions at the rate of many billions per second, but there is no net transport of charge, thus **no current through the wire.**
- **CONNECT A BATTERY!** To cause movement of a charge, there must be a potential difference.
- **There now is** a *net* transport of charge and thus **an electric current through the wire.**

- Electric field creates a force which acts on conduction electrons!
- This forces causes to move electrons and electric current is established.



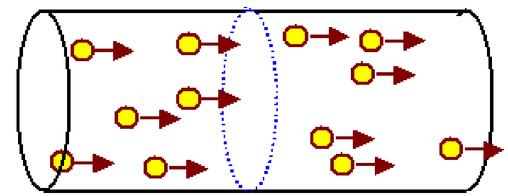


(a) (b)

Fig. 26-1 (*a*) A loop of copper in electrostatic equilibrium. The entire loop is at a single potential, and the electric field is zero at all points inside the copper. (*b*) Adding a battery imposes an electric potential difference between the ends of the loop that are connected to the terminals of the battery. The battery thus produces an electric field within the loop, from terminal to terminal, and the field causes charges to move around the loop. This movement of charges is a current *i*.

- Fig. (a) reminds us, any isolated conducting loop is all at the same potential. No electric field can exist within it or along its surface.
- If we **insert a battery** in the loop, as in Fig. (b),
  - The conducting loop is no longer at a single potential.
  - Electric fields act inside the material making up the loop, exerting forces on internal charges, causing them to move and thus establishing a **current**.

- The **electric current** is the amount of charge per unit time that passes through a plane that pass completely through the conductor.



$$i = \frac{dq}{dt} \qquad \text{(definition of current).}$$

The **SI unit** for current is the **coulomb per second**, or **the ampere (A):**

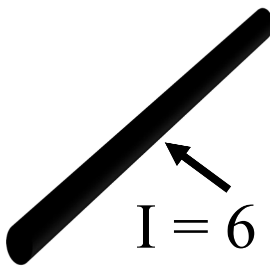
$$1 \text{ ampere} = 1 \text{ A} = 1 \text{ coulomb per second} = 1 \text{ C/s}.$$

$$q = \int dq = \int_0^t i\, dt$$

- The total charge that passes through the plane in a time interval extending from 0 to *t.*

Example: The electric current in a wire is 6 A. How many electrons flow past a given point in a time of 3 s?

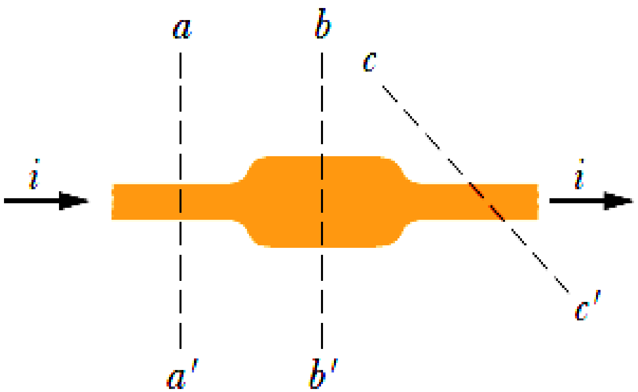
$q = (6 \text{ A})(3 \text{ s}) = 18 \text{ C}$



Recall that: 1 e^- = 1.6 x 10^{-19} C, then convert:

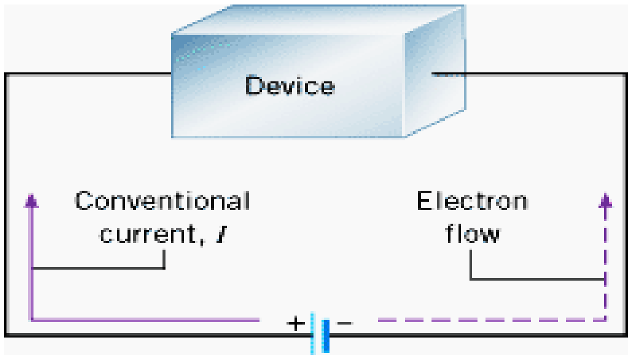
Total number of electrons= 18 C / (1.6x10^{-19} C)

In 3 s: 1.12 x 10^{20} electrons

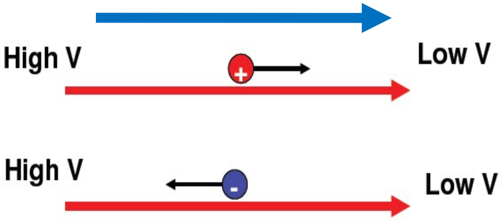


The current is the same in any cross section.

Fig. 26-2 The current i through the conductor has the same value at planes aa', bb', and cc'.



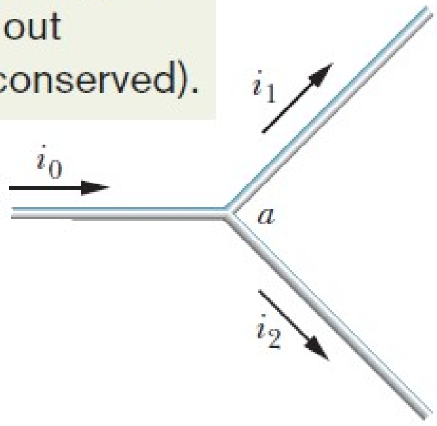
- Under steady-state conditions, the current is the same for planes *aa', bb',* and *cc'* and for all planes that pass completely through the conductor, no matter what their location or orientation.
- The direction of conventional current is always from a point of higher potential (+ terminal) toward a point of lower potential (- terminal).
- Current and positive charge carriers motion are in the same direction (*actual charge carriers are negative and move in the opposite direction*).



A current arrow is drawn in the direction in which positive charge carriers would move, even if the actual charge carriers are negative and move in the opposite direction.

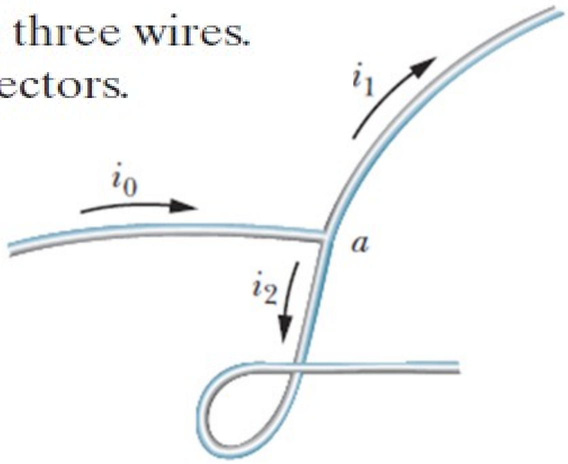
26-2 Electric Current, Conservation of Charge

The current into the junction must equal the current out (charge is conserved).



(a)

Fig. 26-3 The relation $i_0 = i_1 + i_2$ is true at junction a no matter what the orientation in space of the three wires. Currents are scalars, not vectors.



(b)

<u>Kirchoff's Law</u>:
Sum of total current into a junction must equal sum of total current out: *Conservation of Charge!*

- **Alternating Current** (AC)
- Provided by power companies

- **Direct Current** (DC)
- Provided by batteries

CHECKPOINT 1

The figure here shows a portion of a circuit. What are the magnitude and direction of the current i in the lower right-hand wire?

Fill in the blanks.
Think water in hose!

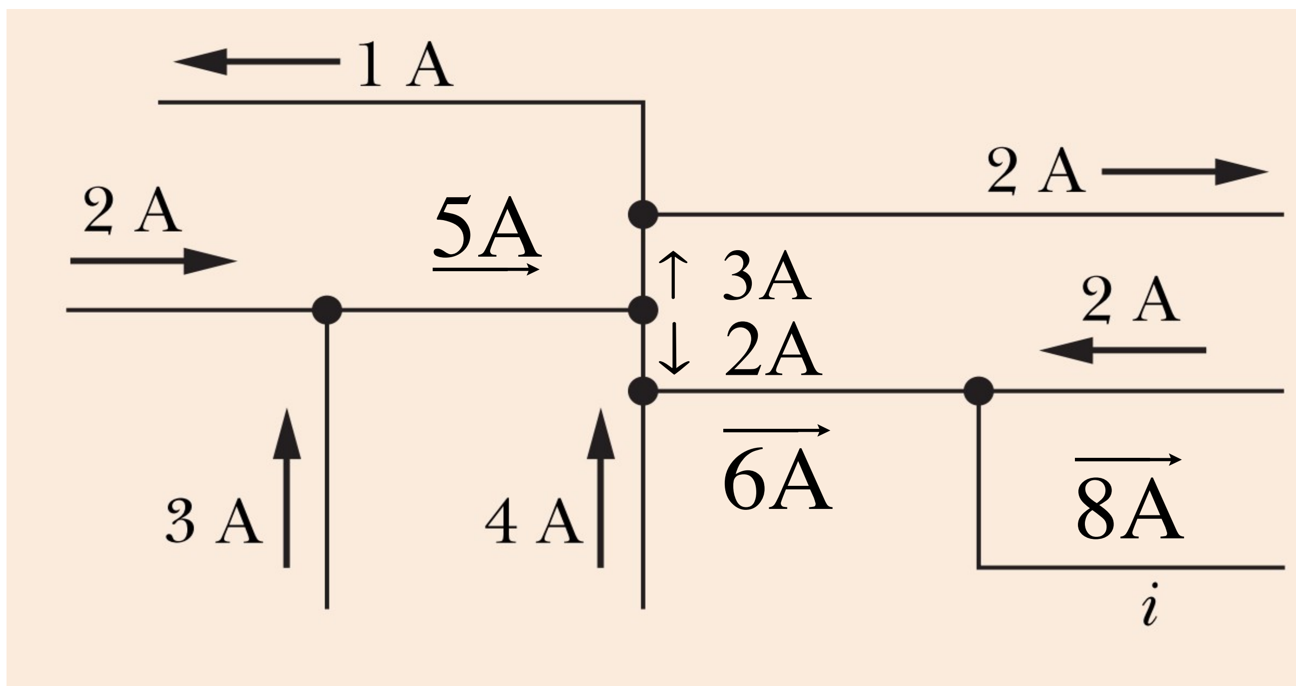
$2 + 3 = 5$

$1 + 2 = 3$

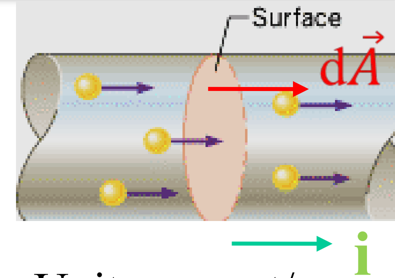
$3 + 2 = 5$

$2 + 4 = 6$

$6 + 2 = 8$



- *Flow of charge through the cross-section* of a conductor at a particular-point is described by **current density, J.**
  - Current density, **J**, is a vector quantity that has both magnitude and direction.
  - Current is uniform across the surface and parallel to dA.



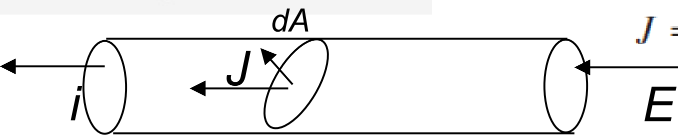
Unit: current/area: A/m^2

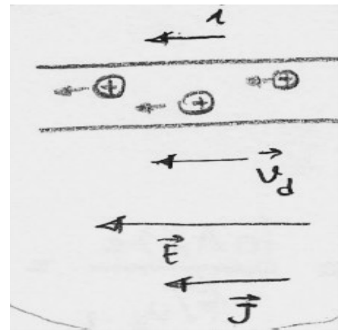
Magnitude of current density

$$i = \int \vec{J} \cdot d\vec{A}.$$

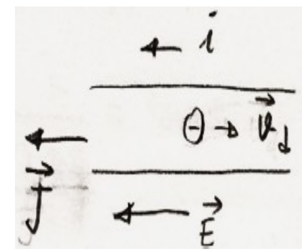
$$i = \int J\,dA = J \int dA = JA$$

$$J = \frac{i}{A},$$





Direction of current density



- It has the same direction as the velocity of moving charges if they are positive and the opposite direction if they are negative.
- Current density can be represented by *streamlines.*
- Current in Section a is equal to current in Section b since charge is conserved. However, the current density changes-it is greater in the narrower conductor; $J_a < J_b$

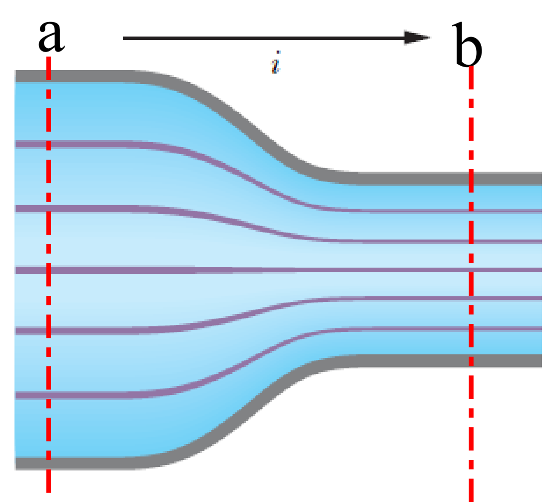


Fig. 26-4 Streamlines representing current density in the flow of charge through a constricted conductor.

26-3 Current Density, Drift Speed

Current is said to be due to positive charges that are propelled by the electric field.

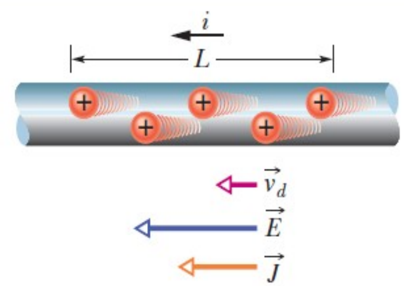


Fig. 26-5 Positive charge carriers drift at speed v_d in the direction of the applied electric field $\vec{E}$. By convention, the direction of the current density $\vec{J}$ and the sense of the current arrow are drawn in that same direction.

Drift speed: v_d :**Velocity at which electrons move in order to establish a current.**

- When a conductor has a current passing through it, the electrons move *randomly*, but they tend to *drift* with a **drift speed *v_d*** in the direction *opposite that of the applied electric* field that causes the current.
- **The drift speed is tiny compared with the speeds in the random motion.**

- In the figure, the drift of positive charge carriers is in the direction of the applied electric field, ***E***. If we assume that
  - these charge carriers all move with the *same drift speed v_d*,
  - the current density ***J*** is *uniform* across the wire's cross-sectional area *A*,
- The number of charge carriers in a length *L* of the wire is *nAL. * Here *n* is the number of carriers per unit volume (**Carrier charge density,** C/m³).
- The total charge of the carriers in the length *L*, each with charge *e*, is then

$$q = (nAL)e.$$

- The total charge moves through any cross section of the wire in the time interval

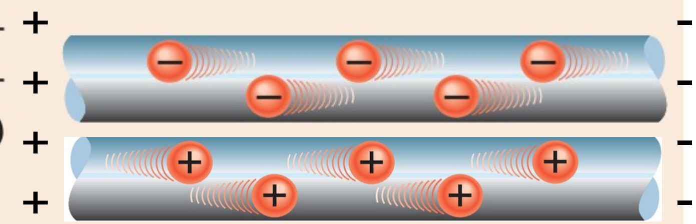
$$t = \frac{L}{v_d}.$$

$$\Rightarrow \quad i = \frac{q}{t} = \frac{nALe}{L/v_d} = nAev_d. \quad \Rightarrow \quad v_d = \frac{i}{nAe} = \frac{J}{ne}$$

$$\Rightarrow \quad \vec{J} = (ne)\vec{v}_d.$$

CHECKPOINT 2

The figure shows conduction electrons moving leftward in a wire. Are the following leftward or rightward: (a) the current i, (b) the current density $\vec{J}$, (c) the electric field $\vec{E}$ in the wire?



All quantities defined in terms of + charge movement!

(a) right $\underset{\rightarrow}{i}$

(b) right $\vec{J} \rightarrow$

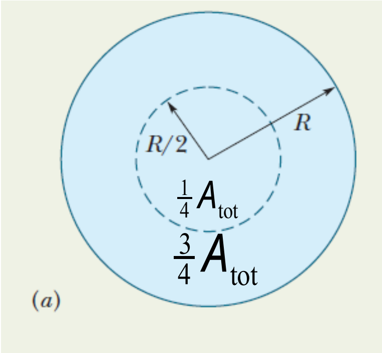
(c) right $\vec{\mathbf{E}} \rightarrow$

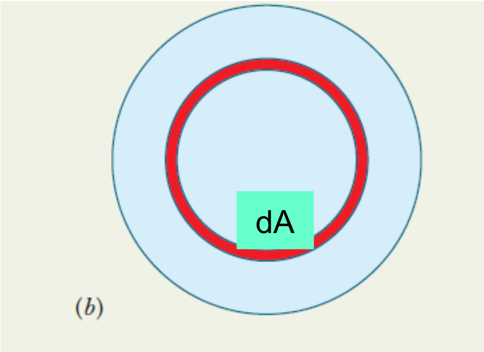
(d) right $\vec{v}_d \rightarrow$

Example, Current Density, Uniform and Nonuniform:

(a) The current density in a cylindrical wire of radius $R = 2.0$ mm is uniform across a cross section of the wire and is $J = 2.0 \times 10^5$ A/m². What is the current through the outer portion of the wire between radial distances $R/2$ and R (Fig. 26-6a)?

Fig. 26-6





Calculations: We want only the current through a reduced cross-sectional area A' of the wire (rather than the entire area), where

$$A' = \pi R^2 - \pi\left(\frac{R}{2}\right)^2 = \pi\left(\frac{3R^2}{4}\right)$$

$$= \frac{3\pi}{4}(0.0020 \text{ m})^2 = 9.424 \times 10^{-6} \text{ m}^2.$$

$$i \sim A \sim R^2$$

$$A' = \frac{3}{4}A_{tot}$$

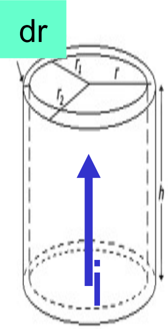
So, we rewrite Eq. 26-5 as

$$i = JA'$$

and then substitute the data to find

$$i = (2.0 \times 10^5 \text{ A/m}^2)(9.424 \times 10^{-6} \text{ m}^2)$$
$$= 1.9 \text{ A}. \quad \text{(Answer)}$$

(b) Suppose, instead, that the current density through a cross section varies with radial distance r as $J = ar^2$, in which $a = 3.0 \times 10^{11}$ A/m⁴ and r is in meters. What now is the current through the same outer portion of the wire?



Calculations: The current density vector $\vec{J}$ (along the wire's length) and the differential area vector $d\vec{A}$ (perpendicular to a cross section of the wire) have the same direction. Thus,

$$\vec{J} \cdot d\vec{A} = J\, dA \cos 0 = J\, dA.$$

We need to replace the differential area dA with something we can actually integrate between the limits $r = R/2$ and $r = R$. The simplest replacement (because J is given as a function of r) is the area $2\pi r\, dr$ of a thin ring of circumference $2\pi r$ and width dr (Fig. 26-6b). We can then integrate

$$i = \int \vec{J} \cdot d\vec{A} = \int J\, dA$$

$$= \int_{R/2}^{R} ar^2\, 2\pi r\, dr = 2\pi a \int_{R/2}^{R} r^3\, dr$$

$$= 2\pi a \left[\frac{r^4}{4}\right]_{R/2}^{R} = \frac{\pi a}{2}\left[R^4 - \frac{R^4}{16}\right] = \frac{15}{32}\pi a R^4$$

$$= \frac{15}{32}\pi(3.0 \times 10^{11} \text{ A/m}^4)(0.0020 \text{ m})^4 = 7.1 \text{ A}.$$

(Answer)

Example, In a current, the conduction electrons move very slowly:

What is the drift speed of the conduction electrons in a copper wire with radius $r = 900\ \mu\text{m}$ when it has a uniform current $i = 17$ mA? Assume that each copper atom contributes one conduction electron to the current and that the current density is uniform across the wire's cross section.

Calculations: Let us start with the third idea by writing

$$n = \begin{pmatrix}\text{atoms}\\ \text{per unit}\\ \text{volume}\end{pmatrix} = \begin{pmatrix}\text{atoms}\\ \text{per}\\ \text{mole}\end{pmatrix}\begin{pmatrix}\text{moles}\\ \text{per unit}\\ \text{mass}\end{pmatrix}\begin{pmatrix}\text{mass}\\ \text{per unit}\\ \text{volume}\end{pmatrix}.$$

The number of atoms per mole is just Avogadro's number N_A $(= 6.02 \times 10^{23}\ \text{mol}^{-1})$. Moles per unit mass is the inverse of the mass per mole, which here is the molar mass M of copper. The mass per unit volume is the (mass) density ρ_{mass} of copper. Thus,

$$n = N_A\left(\frac{1}{M}\right)\rho_{\text{mass}} = \frac{N_A \rho_{\text{mass}}}{M}.$$

Taking copper's molar mass M and density ρ_{mass} from Appendix F, we then have (with some conversions of units)

$$n = \frac{(6.02 \times 10^{23}\ \text{mol}^{-1})(8.96 \times 10^{3}\ \text{kg/m}^3)}{63.54 \times 10^{-3}\ \text{kg/mol}}$$

$$= 8.49 \times 10^{28}\ \text{electrons/m}^3$$

or

$$n = 8.49 \times 10^{28}\ \text{m}^{-3}.$$

Next let us combine the first two key ideas by writing

$$\frac{i}{A} = nev_d.$$

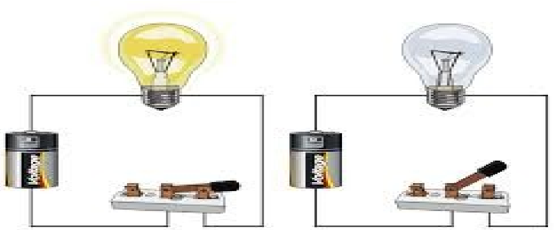
Substituting for A with πr^2 $(= 2.54 \times 10^{-6}\ \text{m}^2)$ and solving for v_d, we then find

$$v_d = \frac{i}{ne(\pi r^2)}$$

$$= \frac{17 \times 10^{-3}\ \text{A}}{(8.49 \times 10^{28}\ \text{m}^{-3})(1.6 \times 10^{-19}\ \text{C})(2.54 \times 10^{-6}\ \text{m}^2)}$$

$$= 4.9 \times 10^{-7}\ \text{m/s}, \qquad \text{(Answer)}$$

which is only 1.8 mm/h, slower than a sluggish snail.

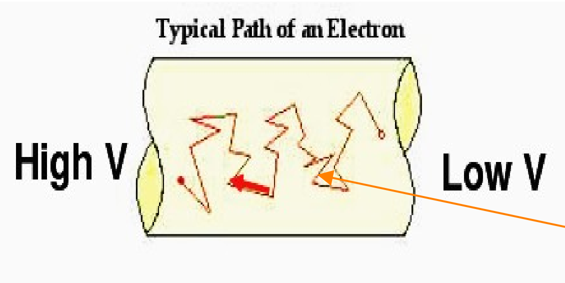


If electrons move so slowly, Why does a light come on the instant You throw the switch?

http://www.phys.lsu.edu/~jdowling/PHYS21132-SP15/lectures/index.html

26-4 Resistance and Resistivity

- **Resistance** btw any two points of a conductor is found by applying a potential difference *V* btw those points and measuring the current *i* that results.



- Electrons are not "completely free to move" in a conductor.
- Electrons moving through a material undergo **many collisions** which hinders their motion.
- The mechanical analog is friction.
- The resistance *R* is then

$$R = \frac{V}{i} \qquad \text{(definition of } R\text{)}.$$

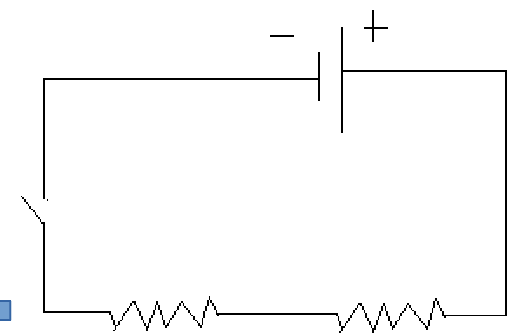
- The SI unit for resistance is the volt per ampere. This has a special name, the **ohm** (symbol Ω):

$$1 \text{ ohm} = 1\ \Omega = 1 \text{ volt per ampere} = 1 \text{ V/A}.$$

Resistance α Potential

- In a circuit diagram, we represent a resistor and a resistance with the symbol -\/\/\/-

Fig. 26-7 An assortment of resistors. The circular bands are color-coding marks that identify the value of the resistance. *(The Image Works)*



Devices specifically designed to have a constant value of *R* are called resistors

The resistivity ρ of a resistor is defined as:

$$\rho = \frac{E}{J} \longrightarrow \vec{E} = \rho \vec{J}.$$

The SI unit for ρ is $\Omega.m$.

The conductivity σ of a material is the reciprocal of its resistivity:

$$\sigma = \frac{1}{\rho} \longrightarrow \vec{J} = \sigma \vec{E}.$$

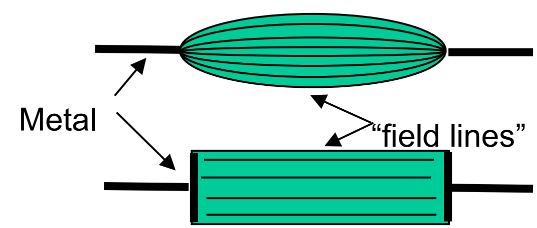
The SI unit for σ is *Siemens/m*.

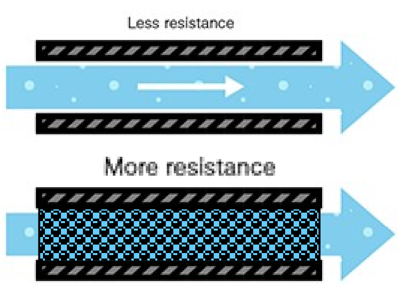
Table 26-1

Resistivities of Some Materials at Room Temperature (20°C)

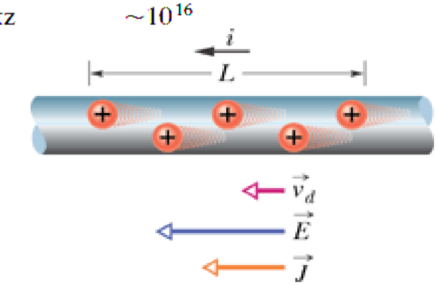
Material	Resistivity, ρ ($\Omega \cdot m$)	Temperature Coefficient of Resistivity, α (K^{-1})
	Typical Metals	
Silver	1.62×10^{-8}	4.1×10^{-3}
Copper	1.69×10^{-8}	4.3×10^{-3}
Gold	2.35×10^{-8}	4.0×10^{-3}
Aluminum	2.75×10^{-8}	4.4×10^{-3}
Manganin[a]	4.82×10^{-8}	0.002×10^{-3}
Tungsten	5.25×10^{-8}	4.5×10^{-3}
Iron	9.68×10^{-8}	6.5×10^{-3}
Platinum	10.6×10^{-8}	3.9×10^{-3}
	Typical Semiconductors	
Silicon, pure	2.5×10^{3}	-70×10^{-3}
Silicon, *n*-type[b]	8.7×10^{-4}	
Silicon, *p*-type[c]	2.8×10^{-3}	
	Typical Insulators	
Glass	$10^{10} - 10^{14}$	
Fused quartz	$\sim 10^{16}$	

These two devices could have the same resistance *R*, when measured on the outgoing metal leads. However, it is obvious that inside of them different things go on.





Put pebbles in the hose, increase resistivity ρ, increase resistance R.



Resistance is a property of an object. Resistivity is a property of a material.

Current is driven by a potential difference.

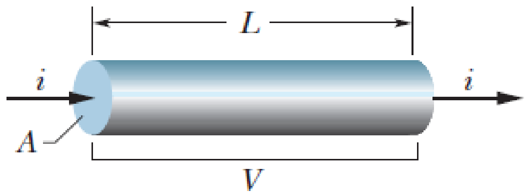


Fig. 26-9 A potential difference V is applied between the ends of a wire of length L and cross section A, establishing a current i.

$$E = V/L \quad \text{and} \quad J = i/A.$$

$$\rho = \frac{E}{J} = \frac{V/L}{i/A}.$$

$$R = \rho \frac{L}{A}.$$

Resistance may vary depending on the **geometry** of the material (length, L and cross-sectional area, A) but **resistivity not**.

Longer → More resistance

Wider → Less resistance

Resistance is a property of an object. Resistivity is a property of a material.

The copper wire has radius *r*. What happens to the Resistance *R* if you:

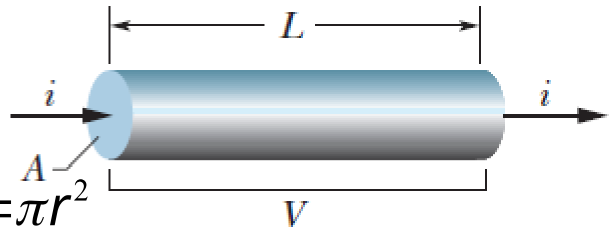
(a)Double the Length? $R \to 2R$

(b)Double the Area? $R \to R/2$

(c)Double the Radius? $R \to R/4$

$$R = \rho \frac{L}{A}.$$

Current is driven by a potential difference.



$A = \pi r^2$

Fig. 26-9 A potential difference V is applied between the ends of a wire of length L and cross section A, establishing a current i.

What happens to the Resistivity ρ if you:

(a)Double the Length? $\rho \to \rho$

(b)Double the Area? $\rho \to \rho$

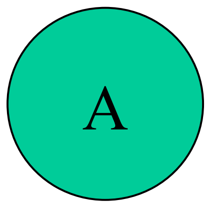
(c)Double the Radius? $\rho \to \rho$

Example: Two conductors are made of the same material and have the same length.

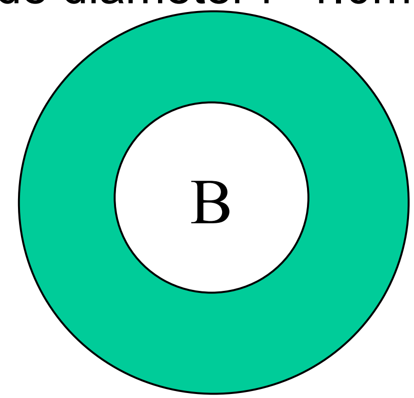
What is the resistance ratio R_A/R_B, measured between their ends?

Conductor *A* is a solid wire of diameter r=1.0mm.

Conductor *B* is a hollow tube of outside diameter 2r=2.0mm and inside diameter r=1.0mm.



$$R=\rho L/A$$



$$A_A=\pi r^2$$

$$A_B=\pi(2r)^2-\pi r^2=3\pi r^2$$

$$R_A/R_B=A_B/A_A=3$$

$L_A=L_B=L$ & ρ Cancels

http://www.phys.lsu.edu/~jdowling/PHYS21132-SP15/lectures/index.html

26-4 Resistance and Resistivity

Example, A material has resistivity, a block of the material has a resistance:

A rectangular block of iron has dimensions 1.2 cm × 1.2 cm × 15 cm. A potential difference is to be applied to the block between parallel sides and in such a way that those sides are equipotential surfaces (as in Fig. 26-8*b*). What is the resistance of the block if the two parallel sides are (1) the square ends (with dimensions 1.2 cm × 1.2 cm) and (2) two rectangular sides (with dimensions 1.2 cm × 15 cm)?

KEY IDEA

The resistance R of an object depends on how the electric potential is applied to the object. In particular, it depends on the ratio L/A, according to Eq. 26-16 ($R = \rho L/A$), where A is the area of the surfaces to which the potential difference is applied and L is the distance between those surfaces.

Calculations: For arrangement 1, we have $L = 15\text{ cm} = 0.15\text{ m}$ and

$$A = (1.2\text{ cm})^2 = 1.44 \times 10^{-4}\text{ m}^2.$$

Substituting into Eq. 26-16 with the resistivity ρ from Table 26-1, we then find that for arrangement 1,

$$R = \frac{\rho L}{A} = \frac{(9.68 \times 10^{-8}\ \Omega\cdot\text{m})(0.15\text{ m})}{1.44 \times 10^{-4}\text{ m}^2} 3$$

$$= 1.0 \times 10^{-4}\ \Omega = 100\ \mu\Omega. \qquad \text{(Answer)}$$

Similarly, for arrangement 2, with distance $L = 1.2$ cm and area $A = (1.2\text{ cm})(15\text{ cm})$, we obtain

$$R = \frac{\rho L}{A} = \frac{(9.68 \times 10^{-8}\ \Omega\cdot\text{m})(1.2 \times 10^{-2}\text{ m})}{1.80 \times 10^{-3}\text{ m}^2}$$

$$= 6.5 \times 10^{-7}\ \Omega = 0.65\ \mu\Omega. \qquad \text{(Answer)}$$

Example

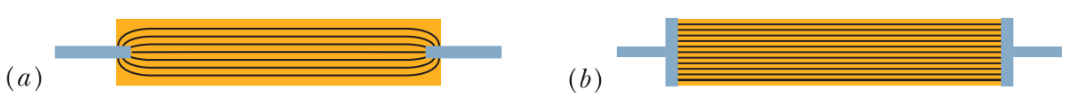


Fig. 26-8 Two ways of applying a potential difference to a conducting rod. The gray connectors are assumed to have negligible resistance. When they are arranged as in (*a*) in a small region at each rod end, the measured resistance is larger than when they are arranged as in (*b*) to cover the entire rod end.

Example, In a current, the conduction electrons move very slowly:

The resistivity of seawater is about 25 $\Omega \cdot \text{cm}$. The charge carriers are chiefly Na^{+} and Cl^{-} ions, and of each there are about $3 \times 10^{20} / \text{cm}^3$. If we fill a plastic tube 2 meters long with seawater and connect a 12-volt battery to the electrodes at each end, what is the resulting average drift velocity of the ions, in cm/s?

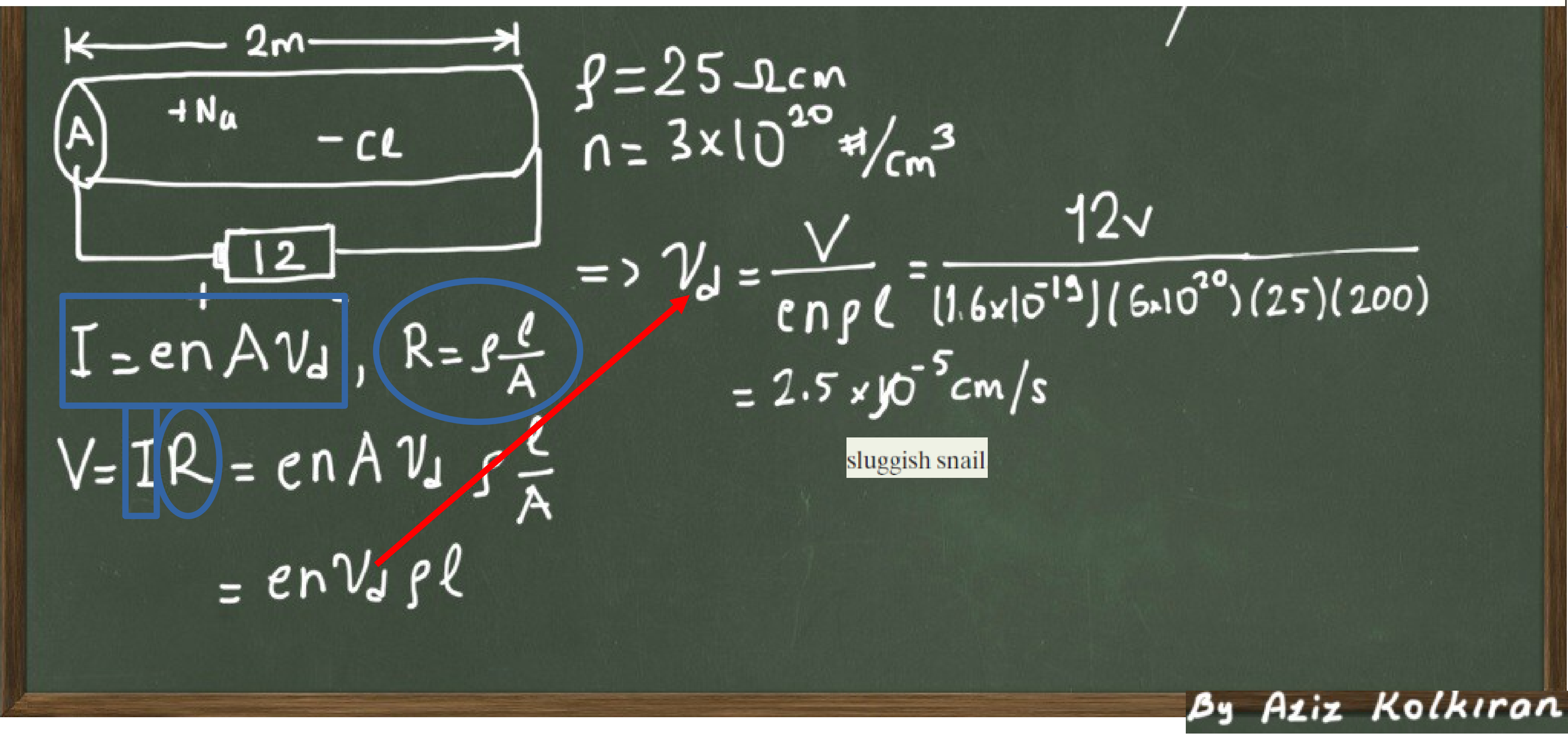
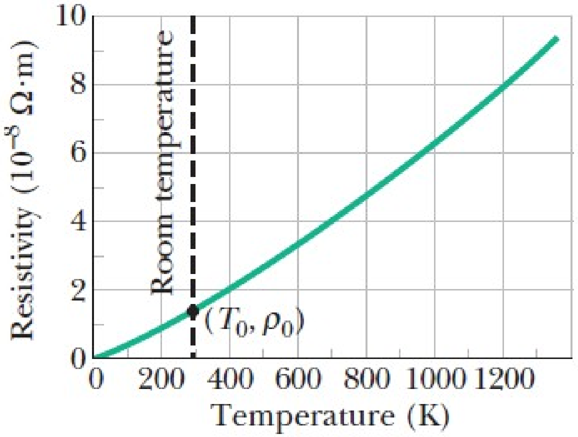


Fig. 26-10 The resistivity of copper as a function of temperature. The dot on the curve marks a convenient reference point at temperature $T_0 = 293$ K and resistivity $\rho_0 = 1.69 \times 10^{-8}\ \Omega \cdot \text{m}$.



Resistivity can depend on temperature.

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0).$$

ρ: resistivity
ρ_o: resistivity at selected reference point
T: temperature
T_o: reference temperature
α: temperature coefficient resistivity

The relation between **Temperature and Resistivity** for copper (for metals in general) is fairly *linear* over a rather broad temperature range.

By Aziz Kolkıran

Ex: Resistance and temperature

If a silver wire has a resistance of $6\,\Omega$ at $20^\circ C$ what resistance will it have at $34\,^\circ C$?

(Temperature coefficient of silver $\alpha = 3.8 \times 10^{-3}\ ^\circ C^{-1}$)

$$R(T) = R(T_0)\left[1 + \alpha (T - T_0)\right]$$

$R(20^\circ C) = 6\,\Omega$

$R(34\,^\circ C) = ?$

$$R(34\,^\circ C) = 6\left[1 + 3.8\times 10^{-3}(34-20)\right]$$

$$= 6,252\ ^\circ C$$

A person with dry skin has a resistance from one arm to the other of about $1 \times 10^5\ \Omega$. When skin is wet, resistance drops to about $1.5 \times 10^3\ \Omega$. (refer to Table 22-1).

TABLE 22–1	
The Damage Caused by Electric Shock	
Current	**Possible Effects**
1 mA	mild shock can be felt
5 mA	shock is painful
15 mA	muscle control is lost
100 mA	death can occur

$R_d = 1.0\times10^5\Omega$

$R_w = 1.5\times10^3\Omega$

$i_m = 1\times10^{-3}A$

(a) What is the minimum voltage placed across the arms that would produce a current that could be felt by a person with dry skin?

100 V

$V_1 = i_m R_d$

(b) What current would the same voltage produce if the person had wet skin?

66.7 mA

$i_1 = V_1/R_w$

(c) What would be the minimum voltage that would produce a current that could be felt when the skin is wet?

1.5 V

$V_2 = i_m R_w$

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Ohm's law is an assertion that the current through a device is *always* directly proportional to the potential difference applied to the device.

A conducting device obeys Ohm's law when the resistance of the device is independent of the magnitude and polarity of the applied potential difference.

A conducting material obeys Ohm's law when the resistivity of the material is independent of the magnitude and direction of the applied electric field.

Georg Simon Ohm (1789-1854)

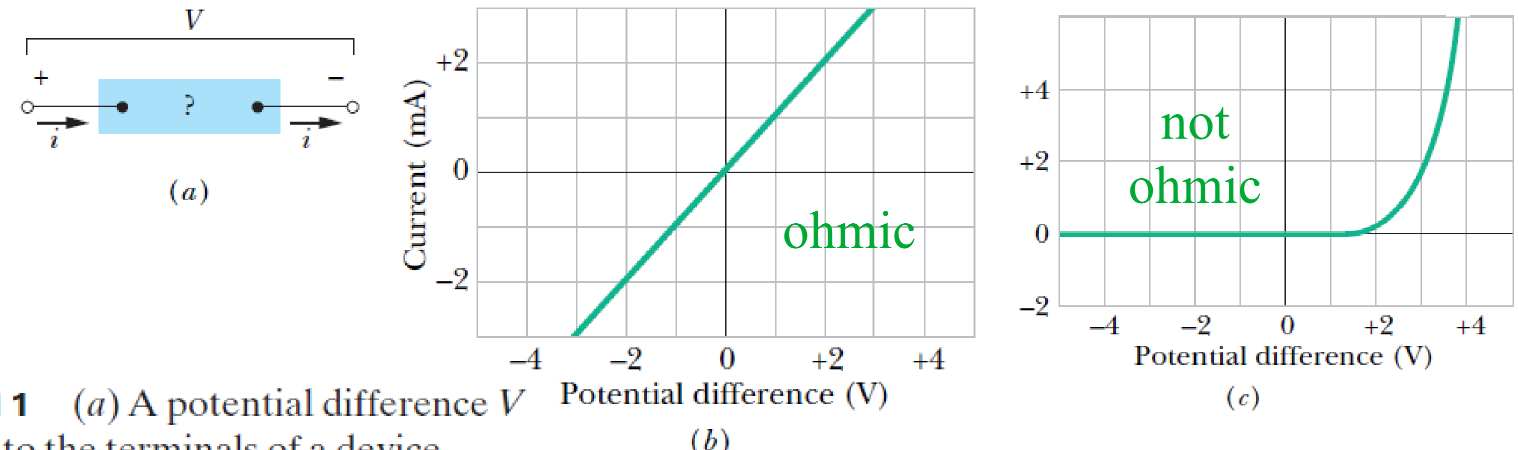

Fig. 26-11 (*a*) A potential difference V is applied to the terminals of a device, establishing a current i. (*b*) A plot of current i versus applied potential difference V when the device is a 1000 Ω resistor. (*c*) A plot when the device is a semiconducting *pn* junction diode.

Most metals, having good conductivity are ohmic.

$J = \sigma E \rightarrow J = \sigma \cdot \frac{\Delta V}{\ell}$ where $J = \frac{I}{A}$

$\frac{I}{A} = \sigma \frac{\Delta V}{\ell}$ ($\sigma = 1/\rho$) $\rightarrow \frac{\rho \cdot \ell}{A} I = \Delta V$ ($\frac{\rho\ell}{A} \sim R$) $\Rightarrow \boxed{\Delta V = IR}$

Macroscopic version of Ohm's law.

CHECKPOINT 4

The following table gives the current i (in amperes) through two devices for several values of potential difference V (in volts). From these data, determine which device does not obey Ohm's law.

Device 1		Device 2	
V	i	V	i
2.00	4.50	2.00	1.50
3.00	6.75	3.00	2.20
4.00	9.00	4.00	2.80

$$V = iR$$

$$R = V / i = \text{constant}$$

```
In[1]:= N[{2/4.5, 3/6.75, 4/9}]
Out[1]= {0.444444, 0.444444, 0.444444}
```

```
In[2]:= N[{2/1.5, 3/2.20, 4/2.8}]
Out[2]= {1.33333, 1.36364, 1.42857}
```

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Example

A voltage of 100V is put over a thick wire of unknown material. The current is measured is $4.5x10^3$ A. The cross section of the wire is $1cm^2$ and the length is 10 m. What material is the cable made of?

$R = V/I = 0.022\ \Omega = \rho\ L/A \rightarrow \rho = 0.022 A/L$

$A = 1cm^2 = 0.0001m^2$ & $L = 10$ m $\rightarrow \rho = 2.2x10^{-7}$ ohm.m ➔ **Lead**

Material	Resistivity (Ohm.m)
Silver	$1.59x10^{-8}$
Gold	$2.44x10^{-8}$
Lead	$22x10^{-8}$
Silicon	640
Quartz	$75x10^{16}$

- It is often assumed that the conduction electrons in a metal move with a single effective speed v_{eff}, and this speed is essentially *independent of the temperature*.
  - For copper, v_{eff} =1.6 x10^6 m/s.
- When we apply an electric field to a metal sample, the electrons modify their random motions slightly and **drift very slowly** with an average drift speed v_d.
  - The *drift speed* in a typical metallic conductor is about 5 x10^{-7} m/s, less than the *effective speed* (1.6 x10^6 m/s) by many orders of magnitude.
- The motion of conduction electrons in an electric field is a combination of the motion due to random collisions and that due to ***E***.
- If an electron of mass *m* is placed in an electric field of magnitude *E*, the electron will experience an acceleration:

$$a = \frac{F}{m} = \frac{eE}{m}.$$

In the average time τ between collisions, the average electron will acquire a drift speed of $v_d = a\tau$. ⇒

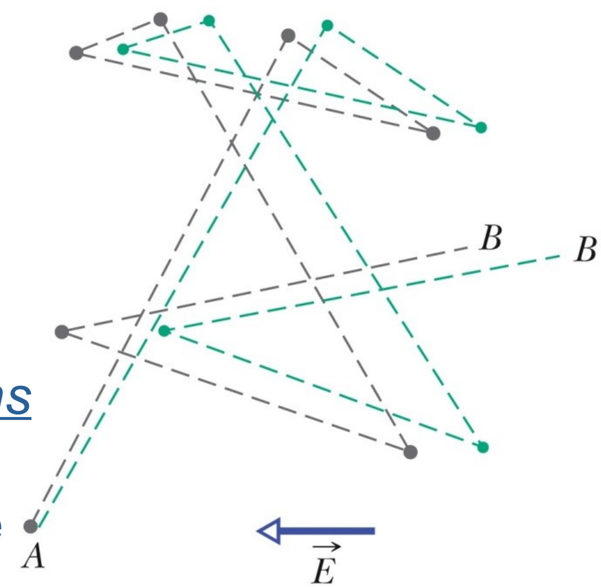
$$v_d = a\tau = \frac{eE\tau}{m}.$$

$$\vec{J} = ne\vec{v}_d \Rightarrow v_d = \frac{J}{ne} = \frac{eE\tau}{m} \Rightarrow E = \left(\frac{m}{e^2 n\tau}\right)J. \Rightarrow \rho = \frac{m}{e^2 n\tau}.$$

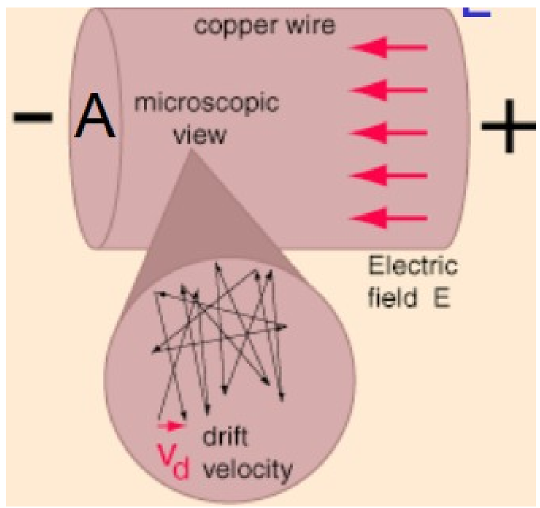
- The assumption that the conduction electrons in a metal are free to move like the molecules in a gas leads to an expression for the *resistivity of a metal*:

$$\rho = \frac{m}{e^2 n \tau}.$$

- Here n is the number of free electrons per unit volume and τ is the mean time between the *collisions of an electron* with the atoms of the metal.
- Metals obey Ohm's law because the mean free time τ is approximately **independent** of the magnitude E of any electric field applied to a metal.



- The gray lines show an electron moving from A to B, making six collisions in route.
- The green lines show what the electron's path might be in the presence of an applied electric field ***E***.
- Note the steady drift in the direction of -***E***.



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Example, Mean Free Time and Mean Free Distance:

(a) What is the mean free time τ between collisions for the conduction electrons in copper?

KEY IDEAS

The mean free time τ of copper is approximately constant, and in particular does not depend on any electric field that might be applied to a sample of the copper. Thus, we need not consider any particular value of applied electric field. However, because the resistivity ρ displayed by copper under an electric field depends on τ, we can find the mean free time τ from Eq. 26-22 ($\rho = m/e^2 n\tau$).

Calculations: That equation gives us

$$\tau = \frac{m}{ne^2\rho}. \tag{26-23}$$

The number of conduction electrons per unit volume in copper is $8.49 \times 10^{28}\ \text{m}^{-3}$. We take the value of ρ from Table 26-1. The denominator then becomes

$$(8.49 \times 10^{28}\ \text{m}^{-3})(1.6 \times 10^{-19}\ \text{C})^2(1.69 \times 10^{-8}\ \Omega\cdot\text{m})$$
$$= 3.67 \times 10^{-17}\ \text{C}^2\cdot\Omega/\text{m}^2 = 3.67 \times 10^{-17}\ \text{kg/s},$$

where we converted units as

$$\frac{\text{C}^2\cdot\Omega}{\text{m}^2} = \frac{\text{C}^2\cdot\text{V}}{\text{m}^2\cdot\text{A}} = \frac{\text{C}^2\cdot\text{J/C}}{\text{m}^2\cdot\text{C/s}} = \frac{\text{kg}\cdot\text{m}^2/\text{s}^2}{\text{m}^2/\text{s}} = \frac{\text{kg}}{\text{s}}.$$

Using these results and substituting for the electron mass m, we then have

$$\tau = \frac{9.1 \times 10^{-31}\ \text{kg}}{3.67 \times 10^{-17}\ \text{kg/s}} = 2.5 \times 10^{-14}\ \text{s}. \quad \text{(Answer)}$$

(b) The mean free path λ of the conduction electrons in a conductor is the average distance traveled by an electron between collisions. (This definition parallels that in Section 19-6 for the mean free path of molecules in a gas.) What is λ for the conduction electrons in copper, assuming that their effective speed v_{eff} is 1.6×10^6 m/s?

KEY IDEA

The distance d any particle travels in a certain time t at a constant speed v is $d = vt$.

Calculation: For the electrons in copper, this gives us

$$\lambda = v_{\text{eff}}\tau \tag{26-24}$$
$$= (1.6 \times 10^6\ \text{m/s})(2.5 \times 10^{-14}\ \text{s})$$
$$= 4.0 \times 10^{-8}\ \text{m} = 40\ \text{nm}. \quad \text{(Answer)}$$

This is about 150 times the distance between nearest-neighbor atoms in a copper lattice. Thus, on the average, each conduction electron passes many copper atoms before finally hitting one.

The battery at the left supplies energy to the conduction electrons that form the current.

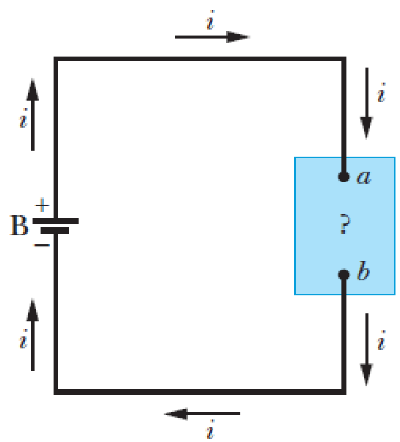

Fig. 26-13 A battery B sets up a current *i* in a circuit containing an unspecified conducting device.

How much work does the battery do to move a small amount of charge dq from b to a?

- A battery "pumps" charges through the resistor (or any device), by producing a potential difference V between points a and b.
- A steady current *i* is produced in the circuit, directed from terminal *a* to terminal *b.*
- The amount of charge ***dq*** that moves from terminals a to b in time interval *dt* is equal to ***i* dt.**
- Its electric potential energy decreases in magnitude by the amount;

$$dU = dq\, V = i\, dt\, V.$$

$dW = –dU = -dq×V = (dq/dt)×dt×V= iV×dt$

The battery "power" is the work it does per unit time:

$P = dW/dt = iV$

- The power *P (rate of electrical energy transfer) dU/dt,* given by

$$P = iV \qquad \text{(rate of electrical energy transfer).}$$

The battery at the left supplies energy to the conduction electrons that form the current.

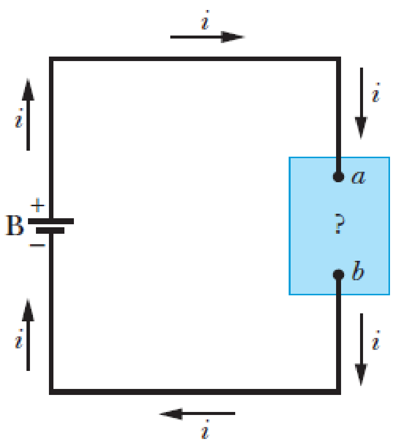

Fig. 26-13 A battery B sets up a current i in a circuit containing an unspecified conducting device.

- This power is also equal **to rate of energy transfer** from battery to device.

$$1\ \mathrm{V\cdot A} = \left(1\,\frac{\mathrm{J}}{\mathrm{C}}\right)\left(1\,\frac{\mathrm{C}}{\mathrm{s}}\right) = 1\,\frac{\mathrm{J}}{\mathrm{s}} = 1\ \mathrm{W}.$$

- *P=iV* is true for the battery pumping charges through any device.
- **If the device follows Ohm's law** (i.e., a resistor), then *V=iR* or *i=V/R* and

$$P = i^2R \qquad \text{(resistive dissipation)}$$

$$P = \frac{V^2}{R} \qquad \text{(resistive dissipation).}$$

Apply only to the transfer of electric potential energy to thermal energy in a device with resistance.

$$P = iV \qquad \text{(rate of electrical energy transfer).}$$

Applies to electrical energy transfers of all kinds.

Example, Rate of Energy Dissipation in a Wire Carrying Current:

You are given a length of uniform heating wire made of a nickel–chromium–iron alloy called Nichrome; it has a resistance R of 72 Ω. At what rate is energy dissipated in each of the following situations? (1) A potential difference of 120 V is applied across the full length of the wire. (2) The wire is cut in half, and a potential difference of 120 V is applied across the length of each half.

KEY IDEA

Current in a resistive material produces a transfer of mechanical energy to thermal energy; the rate of transfer (dissipation) is given by Eqs. 26-26 to 26-28.

Calculations: Because we know the potential V and resistance R, we use Eq. 26-28, which yields, for situation 1,

$$P = \frac{V^2}{R} = \frac{(120\ \text{V})^2}{72\ \Omega} = 200\ \text{W}. \qquad \text{(Answer)}$$

In situation 2, the resistance of each half of the wire is (72 Ω)/2, or 36 Ω. Thus, the dissipation rate for each half is

$$P' = \frac{(120\ \text{V})^2}{36\ \Omega} = 400\ \text{W},$$

and that for the two halves is

$$P = 2P' = 800\ \text{W}. \qquad \text{(Answer)}$$

This is four times the dissipation rate of the full length of wire. Thus, you might conclude that you could buy a heating coil, cut it in half, and reconnect it to obtain four times the heat output. Why is this unwise? (What would happen to the amount of current in the coil?)

Why is this unwise???

$$P = iV \rightarrow 4P = (4i)V$$

Current i increases by 4.
House Circuit Breaker 3A.

$$P = iV = i^2R = V^2/R$$

$$i = P/V = 200W/120V = 1.67A$$

$$\rightarrow 4P/V = 6.67A$$

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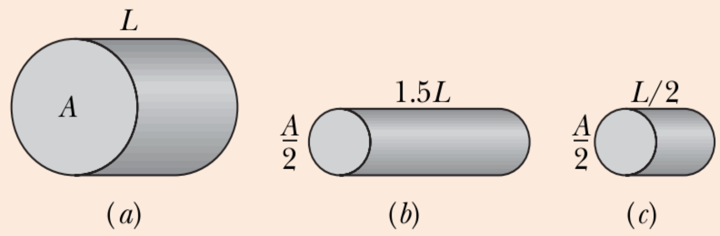
CHECKPOINT 3

The figure here shows three cylindrical copper conductors along with their face areas and lengths. Rank them according to the current through them, greatest first, when the same potential difference V is placed across their lengths.



Rank them according to the power dissipated by them, greatest first, when the same potential difference V is placed across their lengths.

Step I: The resistivity ρ is the same (all three are copper). Find the Resistance R=ρL/A for each case:

$$R_a = \frac{\rho L}{A} \quad R_b = \frac{\rho 3L/2}{A/2} = 3\frac{\rho L}{A} = 3R_a \quad R_c = \frac{\rho L/2}{A/2} = \frac{\rho L}{A} = R_a \quad R_a = R_c < R_b$$

Step II: Rank the current using V=iR or i=V/R with V constant! $i_a = i_c > i_b$

Step III: Rank the power using $P=V^2/R$ since V is same. $P = iV = i^2 R = V^2 / R$

$$P_a = P_c > P_b$$

Ranking is reversed since R is downstairs.

Example: A P=1250 Watt radiant heater is constructed to operate at V=115Volts.

(a) What is the resistance of the heating coil?

(b) What will be the current in the heater?

(c) How much thermal energy is produced in 1.0 hr by the heater?

- Formulas: $P=i^2R=V^2/R$; $V=iR$
- Know P, V; need R to calculate current!

(a) P=1250W; V=115V => $R=V^2/P=(115V)^2/1250W=10.6\ \Omega$

(b) $i=V/R= 115V/10.6\ \Omega=10.8$ A

(c) P=dU/dt => $\Delta U=P\times\Delta t$ = 1.250kW•hr = 1250W × 3600 sec= 4.5 MJ

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26-8 Semiconductors

Table 26-2

Some Electrical Properties of Copper and Silicon

Property	Copper	Silicon
Type of material	Metal	Semiconductor
Charge carrier density, m^{-3}	8.49×10^{28}	1×10^{16}
Resistivity, $\Omega \cdot m$	1.69×10^{-8}	2.5×10^{3}
Temperature coefficient of resistivity, K^{-1}	$+4.3 \times 10^{-3}$	-70×10^{-3}

The resistivity in a conductor is given by:

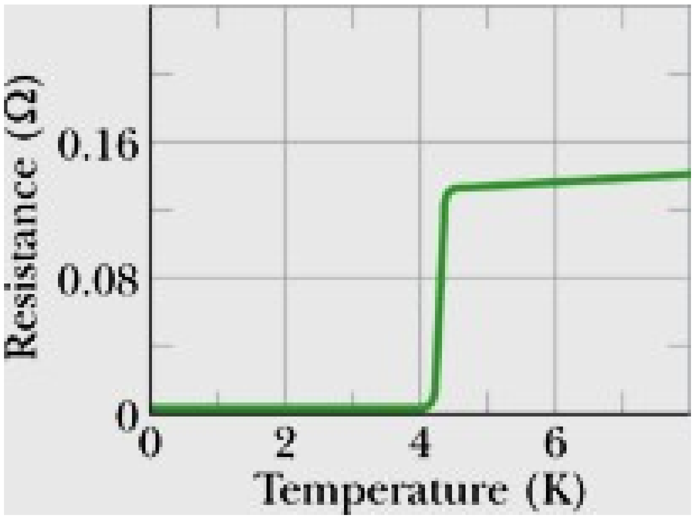
$$\rho = \frac{m}{e^2 n \tau},$$

- A semiconductor is like an insulator except that the energy required to free some electrons is not quite so great.
- In a semiconductor, *n* is small but *increases very rapidly with temperature* as the increased thermal agitation makes more charge carriers available.
  - This causes **a decrease of resistivity with increasing temperature**.

- Pure silicon has a high resistivity and it is effectively an insulator. However, its resistivity can be greatly reduced in a controlled way by adding minute amounts of specific "impurity" atoms in a process called *doping*.
- The process of doping can supply electrons or positive charge carriers that are very loosely held within the material and thus are easy to get moving.
  - Also, by controlling the doping of a semiconductor, one can control the density of charge carriers that are responsible for a current.

26-9 Superconductors

- The **resistivity** of material absolutely disappears at **very low temperatures**.
- This phenomenon is called **superconductivity**.
- It means that charge can flow through a superconducting conductor without losing its energy to thermal energy.



The resistance of mercury drops to zero at a temperature of about 4 K.

A disk-shaped magnet is levitated above a superconducting material that has been cooled by liquid nitrogen. The goldfish is along for the ride. *(Courtesy Shoji Tonaka/International Superconductivity Technology Center, Tokyo, Japan)*

1. (1) During the 4.0 min a 5.0 A current is set up in a wire, how many (a) coulombs and (b) electrons pass through any cross section across the wire's width?

1 (1) $t = 4\ min = 240\ sec$

$i = 5\ A$

i) $i = \frac{q}{t} \rightsquigarrow q = it = (5A)(240\ sec) = 1.2\times10^{3}\ C$

ii) $\# = \frac{q}{e} = \frac{1.2\times10^{3}\ C}{1.6\times10^{-19}\ C} = 7.5\times10^{21}\ e^{-}s$

2. (11) What is the current in a wire of radius R=3.40 mm if the magnitude of the current density is given by (a) $J_a=J_0 r/R$ and (b) $J_b= J_0(1-r/R)$, in which r is the radial distance and $J_0 = 5.50\times10^4$ A/m^2? (c) Which function maximizes the current density near the wire's surface?

2(11) $R = 3.4\times10^{-3}$ m i) $J_i = J_0 r/R$ $i = \int \vec{J}\cdot d\vec{A} = \int J\,dA = \int (J_0 r/R)(2\pi r\,dr) = \int_0^R \frac{J_0 2\pi}{R} r^2 dr$

$\frac{i}{A} = J \rightsquigarrow J(r)$

$J_0 = 5.5\times10^4$ A/m^2

$= \frac{J_0 2\pi}{R} \frac{r^3}{3}\Big|_0^R = J_0 \frac{2\pi}{3} R^2 = (5.5\times10^4 \frac{A}{m^2}) \frac{2}{3}\pi (3\times10^{-3} m)^2$

$= 1.33\,A$

ii) $J_{ii} = J_0(1 - r/R) \rightarrow i = \int_0^R J_0(1-\frac{r}{R}) 2\pi r\,dr = J_0 2\pi\left(\frac{r^2}{2} - \frac{r^3}{3R}\right)\Big|_0^R = J_0 2\pi\left(\frac{R^2}{2} - \frac{R^2}{3}\right) = J_0 2\pi \frac{R^2}{6}$

$= 0.666\,A$

iii) when $r\to R$: $J_i \to J_0$, $J_{ii} \to 0$; when $r \to 0$: $J_i \to 0$, $J_{ii} \to J_0$ } so near the surface $(r\to R)$ J_i maximizes

3. (18) A wire 4.00 m long and 6.00 mm in diameter has a resistance of 15.0 mΩ. A potential difference of 23.0 V is applied between the ends. (a) What is the current in the wire? (b) What is the magnitude of the current density? (c) Calculate the resistivity of the wire material.

3(18) $L = 4\,m$

$r = 3\times10^{-3}\,m$

$R = 15\times10^{-3}\,\Omega$

$V = 23\,V$

i) $i = ?$ $\quad i = V/R = \frac{23\,V}{15\times10^{-3}\,\Omega} = 1.53\times10^{3}\,A$

ii) $J = ?$ $\quad J = i/A = \frac{1.53\times10^{3}\,A}{\pi(3\times10^{-3}\,m)^2} = 5.41\times10^{7}\,A/m^2$

iii) $\rho = ?$ $\quad R = \rho\frac{L}{A} \rightarrow \rho = \frac{RA}{L} = \frac{15\times10^{-3}\,\Omega\;\pi(3\times10^{-3}\,m)^2}{4\,m} = 10.6\times10^{-8}\,\Omega m$

4. (23) When 115 V is applied across a wire that is 10 m long and has a 0.30 mm radius, the magnitude of the current density is $1.4\mathrm{x}10^{4}$ $\mathrm{A/m^2}$. Find the resistivity and conductivity of the wire.

(23) $V = 115\,V$

$L = 10\,m$

$r = 0.3\times10^{-3}\,m$

$J = 1.4\times10^{4}\,A/m^2$

$\rho = ?$

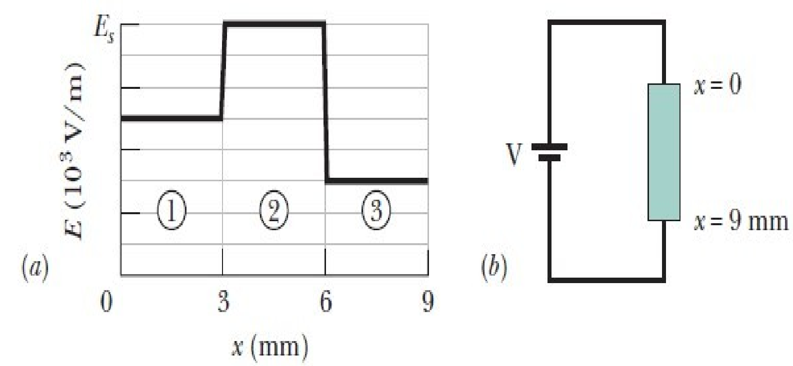
$\sigma = ?$

$E = \rho J$ & $E = \dfrac{V}{L}$

$$\rightarrow \rho = \frac{V/L}{J} = \frac{115\,V/10\,m}{1.4\times10^{4}\,A/m^2} = 8.2\times10^{-4}\;\Omega\cdot m$$

$$\sigma = 1/\rho = 1217.4\,(\Omega m)^{-1}$$

5. (24) Figure gives the magnitude E(x) of the electric fields that have been set up by a battery along a resistive rod of length 9.00 mm. The vertical scale is set by $E_s = 4.00\times10^3$ V/m.



The rod consists of three sections of the same material but with different radii. (The schematic diagram of Fig. 26-24b does not indicate the different radii.) The radius of section 3 is 2.00 mm. What is the radius of (a) section 1 and (b) section 2

4 (24)
- A resistive rod of length 9.0 mm
- Three sections
- $R_3 = 2.00$ m
R_1 & R_2 = ?

Same material → same resistivity & $\vec{E} = \rho \vec{J} \rightarrow E \propto J$
Same current is at each segment (charge conservation)
$i_1 = i_2 = i_3$ { $J_1A_1 = J_2A_2 = J_3A_3$ | $E_1 \sim 2.5 \leftarrow J_1$; $E_2 \sim 4 \leftarrow J_2$; $E_3 \sim 1.5 \leftarrow J_3$

→ $2.5\pi R_1^2 = 4\pi R_2^2$ | $R_3 = 0.02$ m
$1.5\pi R_3^2 = 4\pi R_2^2$
$2.5\pi R_1^2 = 1.5\pi R_3^2$

$R_1 = 1.55$ mm
$R_2 = 1.22$ mm

6. (47) A heating element is made by maintaining a potential difference of 75.0 V across the length of a Nichrome wire that has a 2.60×10^{-6} m^2 cross section. Nichrome has a resistivity of 5.00×10^{-7} Ωm.

(a) If the element dissipates 5000W, what is its length?

(b) If 100 V is used to obtain the same dissipation rate, what should the length be?

9(47)

$V = 75.0\,V$ on wire

$A = 2.60 \times 10^{-6}\,m^2$ cross section

$\rho = 5.00 \times 10^{-7}\,\Omega m$

(i) $\ell = ?$ if $P = 5000W$

$$P = \frac{V^2}{R} = \frac{V^2}{\rho \ell / A} \Rightarrow \ell = \frac{AV^2}{\rho P} = \frac{(2.60 \times 10^{-6}\,m^2)(75.0\,V)^2}{5.00 \times 10^{-7}\,\Omega m\,(5000W)}$$

V^2/R

$\Rightarrow \boxed{\ell = 5.85\,m}$

(ii) $\ell = ?$ if $P = 5000W$ & $V = 100V$

$$\frac{(75\,V)^2}{(100\,V)^2} = \frac{(5.85\,m)}{\ell'} \quad \ell' = (5.85\,m)\frac{(100V)^2}{(75V)^2}$$

$\Rightarrow \boxed{\ell' = 10.4\,m}$

7. (53) A 120 V potential difference is applied to a space heater that dissipates 500 W during operation.

(a) What is its resistance during operation?

(b) At what rate do electrons flow through any cross section of the heater element?

11 (53)

$V = 120\ V$

$P = 500\ W$

i) $R = ?$ $P = iV$; $P = V^2/R$; $P = i^2 R$

$\rightarrow R = \frac{V^2}{P} = \frac{(120\ V)^2}{500\ W} = 28.8\ \Omega$

ii) $\frac{i}{e} \equiv$ # of e's in time interval ; $\frac{P}{Ve} = \frac{500\ W}{(120\ V)(1.602\times10^{-19}\ C)} = 2.60\times10^{19}\ 1/s$

$\rightsquigarrow \frac{W}{VC} \rightarrow \frac{AV}{VC} \sim \frac{C}{\Delta t\, C} \sim 1/s$

İZMİR KATİP ÇELEBİ ÜNİVERSİTESİ

Current

•The electric current i in a conductor is defined by

$$i = \frac{dq}{dt}.$$ **Eq. 26-1**

Current Density

•Current is related to current density by

$$i = \int \vec{J} \cdot d\vec{A},$$ **Eq. 26-4**

Drift Speed of the Charge Carriers

•Drift speed of the charge carriers in an applied electric field is related to current density by $\vec{J} = (ne)\vec{v}_d$, **Eq. 26-7**

Resistance of a Conductor

•Resistance R of a conductor is defined by

$$R = \frac{V}{i}$$ **Eq. 26-8**

•Similarly the resistivity and conductivity of a material is defined by

$$\rho = \frac{1}{\sigma} = \frac{E}{J}$$ **Eq. 26-10&12**

•Resistance of a conducting wire of length L and uniform cross section is

$$R = \rho \frac{L}{A}$$ **Eq. 26-16**

Change of ρ with Temperature

•The resistivity of most material changes with temperature and is given as

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0).$$ **Eq. 26-17**

Ohm's Law

•A given device (conductor, resistor, or any other electrical device) obeys Ohm's law if its resistance R (defined by **Eq. 26-8** as V/i) is independent of the applied potential difference V.

Resistivity of a Metal

•By assuming that the conduction electrons in a metal are free to move like the molecules of a gas, it is possible to derive an expression for the resistivity of a metal:

$$\rho = \frac{m}{e^2 n \tau}.$$ **Eq. 26-22**

Power

•The power P, or rate of energy transfer, in an electrical device across which a potential difference V is maintained is

$$P = iV$$ **Eq. 26-26**

•If the device is a resistor, we can write

$$P = i^2 R = \frac{V^2}{R}$$ **Eq. 26-27&28**