Lecture 4

Probability II

Lecture Information

Ceng272 *Statistical Computations* at March 8, 2010

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Contents

Additive Rules I

- **Theorem 2.10**:
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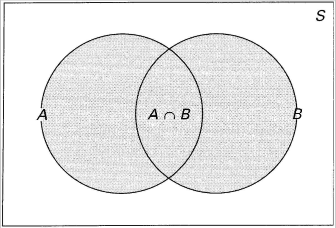

Figure: Additive rule of probability.

Additive Rules II

- **Corollary 2**:

  If $A_1, A_2, \ldots A_n$, are mutually exclusive, then

$$P(A_1 \cup A_2 \cup \ldots \cup A_n) = P(A_1) + P(A_2) + \ldots + P(A_n)$$

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- **Corollary 3**:

If $A_1, A_2, \ldots A_n$, , is a partition of a sample space S, then

$$P(A_1 \cup A_2 \cup \ldots \cup A_n) = P(A_1) + P(A_2) + \ldots + P(A_n)$$

$$= P(S) = 1$$

Additive Rules III

- **Theorem 2.11**: (an extension of Theorem 2.10)

For three events A, B, and C,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C)$$
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- **Theorem 2.12**:

  If A and A' are complementary events, then

  $$P(A) + P(A') = 1$$

Proof : Since $A \cup A' = S$ and $A \cap A' = \varnothing$, then

$$1 = P(S) = P(A \cup A') = P(A) + P(A')$$

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  - What is the probability that a cable selected randomly is too large?
    Let M be the event that a cable meets spec. Let S and L be the events that the cable is too small and too large, respectively. Then

$$P(M) = 0.99 \textit{ and } P(S) = P(L) = (1 - 0.99)/2 = 0.0005$$

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  - What is the probability that a cable selected randomly is larger than 1990 mm?

    $$P(X \geq 1990) = 1 - P(S) = 0.995$$

    where X is the length of a randomly selected cable.

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- **Definition 2.9**:

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

provided $P(A) > 0$

Conditional Probability II

- **Example**: Our sample space S is the population of adults in a small town. They can be categorized according to gender and employment status (see Table 1).

Table: Categorized adult population in a small town.

	Employed	Unemployed	Total
Male	460	40	500
Female	140	260	400
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$$P(M|E) = \frac{460}{600} = \frac{23}{30}$$

$$P(M|E) = \frac{n(E \cap M)/n(S)}{n(E)/n(S)} = \frac{P(E \cap M)}{P(E)} = \frac{\frac{460}{900}}{\frac{600}{900}} = \frac{23}{30}$$

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$$P(D|A) = \frac{P(D \cap A)}{P(A)} = \frac{0.78}{0.82} = 0.95$$

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- **Definition 2.10**:

  Two events A and B are said to be **independent** if and only if

  $$P(B|A) = P(B) \text{ or } P(A|B) = P(A).$$

  Otherwise, A and B are **dependent**.

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$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{1/52}{4/52} = \frac{13}{52} = \frac{1}{4} \text{ and } P(B) = \frac{13}{52} = \frac{1}{4}$$

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  - Since $P(B|A) = P(B)$, these two events are independent.

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- What is the probability that both fuses are defective?
  - Event A: the first fuse is defective
  - Event B: the second fuse is defective. Hence,
  $$P(A \cap B) = P(A) * P(B|A) = \frac{1}{4} * \frac{4}{19} = \frac{1}{19}$$

Multiplicative Rules II

- **Example 2.36**: One bag contains 4 white balls and 3 black balls. A second bag contains 3 white balls and 5 black balls.

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- **Example 2.36**: One bag contains 4 white balls and 3 black balls. A second bag contains 3 white balls and 5 black balls.
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- One ball is drawn from the first bag and placed unseen in the second bag. What is the probability that a ball now drawn from the second bag is black?
- Solution: Let B_1, B_2, and W_1 represent, respectively, the drawing of a black ball from bag 1, a black ball from bag 2, and a white ball from bag 1.

$$p[(B_1 \cap B_2) \cup (W_1 \cap B_2)] = P(B_1 \cap B_2) + P(W_1 \cap B_2)$$

$$= P(B_1)P(B_2|B_1) + P(W_1)P(B_2|W_1)$$

$$= \frac{3}{7} * \frac{6}{9} + \frac{4}{7} * \frac{5}{9} = \frac{38}{63}$$

Multiplicative Rules III

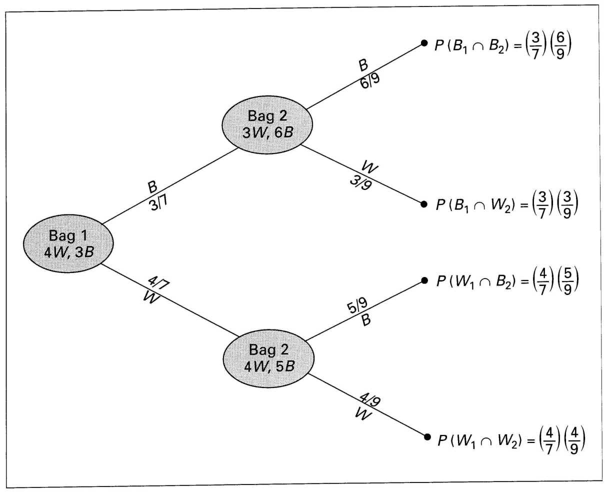

Figure: Tree diagram for Example 2.36.

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  Two events A and B are (statistically or probabilistically) independent if and only if

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- **Solution**: Let A and B represent the respective evens that the fire engine and the ambulance are available. Then

  $$P(A \cap B) = P(A)P(B) = 0.98 * 0.92 = 0.9016.$$

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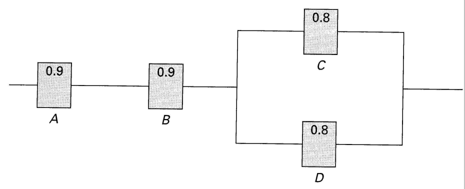

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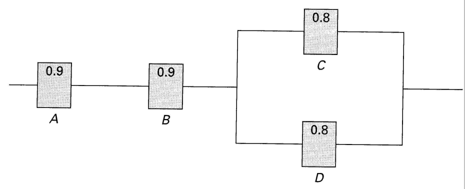

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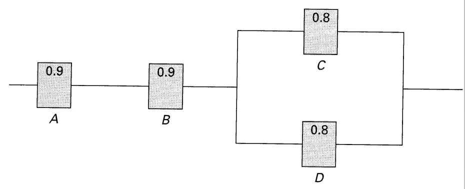

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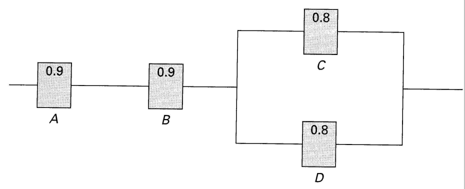

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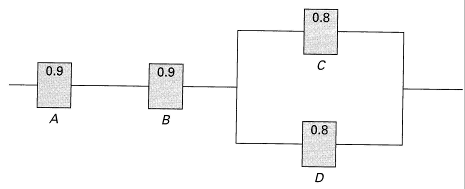

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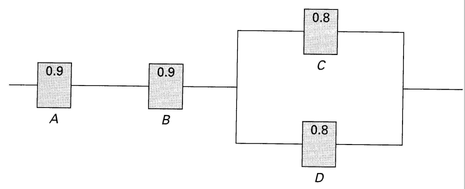

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$$P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2|A_1)P(A_3|A_1 \cap A_2)$$

$$= \frac{2}{52} * \frac{8}{51} * \frac{12}{50} = \frac{8}{5525}$$

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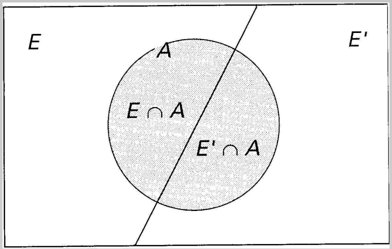

Figure: Venn diagram for the events A, E, and E'.

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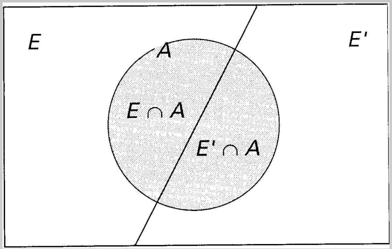

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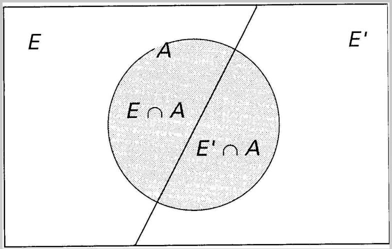

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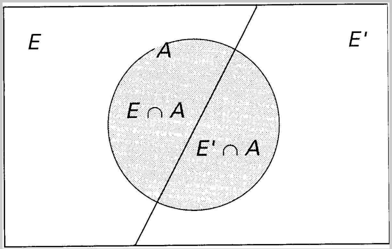

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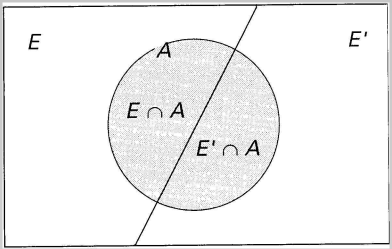
  - Find the probability of the event A that individual selected is a member of the Rotary Club.



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Bayes'Rules II

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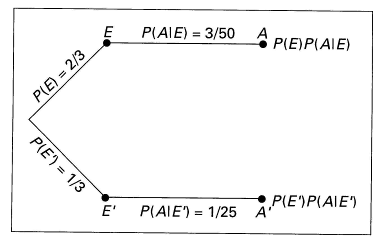

Figure: Tree diagram for the data.

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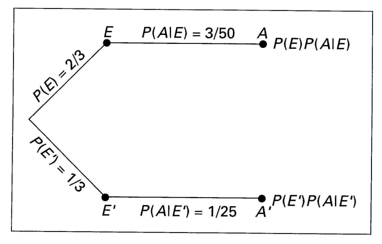

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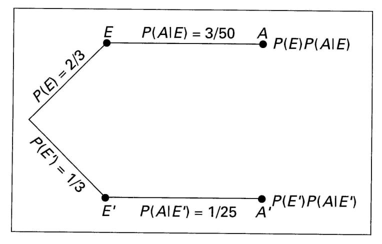
$$\bullet A = (E \cap A) \cup (E' \cap A)$$

$$\bullet P(A) = P[(E \cap A) \cup (E' \cap A)]$$

$$= P(E \cap A) + P(E' \cap A)$$

$$= P(E)P(A|E) + P(E')P(A|E')$$



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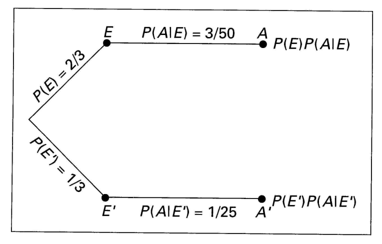
$$\bullet P(E) = \frac{600}{900} = \frac{2}{3},\ P(A|E) = \frac{36}{600} = \frac{3}{50}$$

$$\bullet P(E') = \frac{1}{3}, P(A|E) = \frac{12}{300} = \frac{1}{25}$$

$$\bullet P(A) = \frac{2}{3} * \frac{3}{50} + \frac{1}{3} * \frac{1}{25} = \frac{4}{75}$$



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Bayes'Rules III

- **Theorem 2.16**: (**Theorem of total probability** or **rule of elimination**)

If the events $B_1, B_2, \ldots, B_k$ constitute a partition of the sample space S such that $P(B_i) \neq 0$ for $i = 1, 2, \ldots, k$, then for any event A of S,

$$P(A) = \sum_{i=1}^{k} P(B_i \cap A) = \sum_{i=1}^{k} P(B_i)P(A|B_i)$$

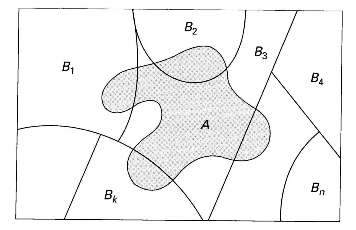

Figure: Partitioning the sample space S.

Bayes'Rules IV

- **Example 2.41**: In a certain assembly plant, three machines, B_1, B_2 and B_3 make 30%, 45% and 25%, respectively, of the products.

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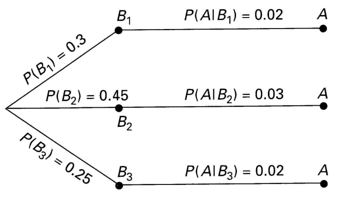

Figure: Tree diagram for Example 2.41.

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- **Theorem 2.17**: (Bayes'Rule)

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$$= \frac{0.005}{0.006 + 0.0135 + 0.005} = \frac{10}{49}$$

Bayes'Rules VII

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- If a random product was observed and found to be defective, which plan was most likely used and thus responsible?
- **Solution**: $P(P_1) = 0.3, (P_12) = 0.2, (P_1) = 0.5$

$$P(P_i|D) = \frac{P(P_i)P(D|P_i)}{\sum_{i=1}^{3} P(P_i)P(D|P_i)} = \frac{(0.30)(0.01)}{(0.3)(0.01) + (0.20)(0.03) + (0.50)(0.02)} = \frac{0.003}{0.019}$$

$$P(P_1|D) = 0.158,\ P(P_2|D) = 0.316,\ P(P_3|D) = 0.526.$$